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Collusion and group lending with adverse selection

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Abstract

In an environment with correlated returns, this paper characterizes optimal lending contracts when the bank faces adverse selection and borrowers have limited liability. Group lending contracts are shown to be dominated by revelation mechanisms which do not use the ex post observability of the partners' performances. However, when collusion between borrowers under complete information is allowed, group lending contracts are optimal in the class of simple revelation mechanisms (which elicit only the borrower's own private information) and remain useful with extended revelation mechanisms.

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1. Introduction

The development of group lending through the Grameen Bank and similar institutions has attracted the interest of all those who believe that lending to the poor is a necessary step to exit the vicious circles of underdevelopment. The empirical evaluation of the success of these new ways of lending to entrepreneurs who have no collateral is still subject to debates (see Khandker et al., 1995; Morduch, 1999; Pitt and Khandker, 1996).

Theorists have proposed various explanations for the new opportunities provided by group lending (see Ghatak and Guinnane, 1999 for a review). In this paper, we restrict our attention to group lending as an instrument to improve discrimination between entrepreneurs of different types (adverse selection).

Ghatak (2000) and Armendariz de Aghion and Gollier (2000) have argued that group lending triggers a peer selection effect among entrepreneurs who know each other. For

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independent types, they show how the knowledge of the types in the group which vary with the different regroupings (for example, in a group of two: two good types or two bad types or one good and one bad type) makes discrimination possible. When entrepreneurs do not know each other, with independent types, group lending brings no improvement (Laffont and N'Guessan, 2000).

In this paper, we propose a simple model to study the role of group lending in discrimination when collusion between borrowers is possible.

We consider exogenously fixed potential pairs of ex ante identical entrepreneurs who carry projects with correlated returns. Each entrepreneur, when he discovers his type, revises his beliefs about the type of his partner, but he does not observe his partner's type. At this point in time, he cannot switch to another partner and draw again his type. When correlation becomes perfect, we have the situation where agents know each other.

Through this modeling, we leave aside the issue of endogenous regrouping to focus on two questions: first, what is the relative power of group lending (for which a successful entrepreneur's repayment depends on the success or failure of his partner) in the class of all possible lending mechanisms? Second, what are the optimal collusion-proof lending contracts and how do the group lending contracts perform from the point of view of collusion?

The model with correlated types is presented in Section 2. The optimal individually incentive-compatible contracts are obtained in Section 3. The place of individual contracts and group lending contracts in the class of individually incentive-compatible mechanisms is explained in Section 4. Section 5 shows that the group lending contracts are in fact optimal when a certain type of group incentive constraints are taken into account. Section 6 considers more general revelation mechanisms and shows that group lending contracts remain useful in this context. Section 7 concludes.

2. The model

There is a continuum of pairs of entrepreneurs, each entrepreneur being associated with a good or a bad project. A good (resp. bad) project returns h when it is successful, i.e. with probability $\bar{p}$ (resp. $\underline{p}$ with $\bar{p}>\underline{p}$), for one unit of investment. For simplicity we consider only projects of size 1.

A pair of entrepreneurs represents a local set of investment opportunities. For simplicity again, we take the case of a group of two entrepreneurs, but, at the cost of more complex notation, it could be a group of any size. However, the size of the group is here exogenous, and we do not raise the issue of the optimal number of entrepreneurs in a group.

Let p_i in $\{\underline{p}, \bar{p}\}$ be the type of entrepreneur i's project, or, for brevity, the type of the entrepreneur. It is private information of the entrepreneur.[1] The types in a pair, (p_1, p_2), are jointly distributed according to the distribution function

$$\pi_{11} = \Pr(p_1 = \underline{p} \text{ and } p_2 = \underline{p})$$

[1] We consider a case of pure adverse selection. It would certainly be interesting to extend the model to situations where the verifiability of the level of production is an issue and enforcement of contracts is imperfect.

$$\pi_{12} = \Pr(p_1 = \underline{p} \text{ and } p_2 = \bar{p}) = \pi_{21} = \Pr(p_1 = \bar{p} \text{ and } p_2 = \underline{p})$$

$$\pi_{22} = \Pr(p_1 = \bar{p} \text{ and } p_2 = \bar{p}).$$

Let $\rho = \pi_{11}\pi_{22} - \pi_{12}\pi_{21}$, be a measure of the correlation of types that we assume to be positive ($\rho > 0$). The positive correlation of returns is the most natural one, as the productivity of investments is often affected by common shocks such as weather or local business conditions.

Entrepreneurs are risk neutral[2] and have no wealth. They must borrow to invest and they can only reimburse their loan if their project is successful. The lender is a monopolistic bank which has a cost of funds r and which maximizes expected profit. We assume that under complete information, all loans are socially valuable because the expected profit from a bad type project is greater than the entrepreneur's opportunity cost $\underline{p}h - r - u > 0$, when u is the status quo utility level of an entrepreneur outside the relationship with the bank. Efficiency calls for all projects to be financed.[3]

3. Optimal contracts

The bank considers the natural groups of entrepreneurs which are the pairs of entrepreneurs with correlated projects and exploits the fact that the structure of correlation is common knowledge. For notational convenience let us refer to type $\underline{p}$ (resp. $\bar{p}$) as type 1 (resp. 2). From the revelation principle, there is no loss of generality in restricting the offer of contracts to two four-uples, $(x_{11}, x_{12}, x_{21}, x_{22})$, $(y_{11}, y_{12}, y_{21}, y_{22})$, where x_{ij} is the repayment of an entrepreneur who has announced that he is of type i when his partner has announced that he is of type j, and when both have succeeded; similarly y_{ij} is the repayment of a successful entrepreneur when his partner has not succeeded, and with the same announcements.[4]

At a truthful Bayesian Nash equilibrium, the participation and incentive constraints of type $\underline{p}$ and $\bar{p}$ are, respectively, (see Appendix A):

$$\pi_{11}(h - \underline{p}x_{11} - (1 - \underline{p})y_{11}) + \pi_{12}(h - \bar{p}x_{12} - (1 - \bar{p})y_{12}) \geq \frac{u}{\underline{p}}(\pi_{11} + \pi_{12}) \quad (1)$$

[2] To assume risk neutrality may appear inappropriate in this context. However, the assumption of limited liability at zero wealth will play a role similar to risk aversion given that there is no ex post moral hazard dimension in the entrepreneurs' activity.

[3] In our framework with a monopolistic bank, it is the interesting case to consider. If only the good type projects were socially valuable, the bank would offer contracts which are only accepted by the good type entrepreneurs and would leave them no rent. Efficiency would always be achieved. Note also that with our stochastic structure, competition of banks would always lead to efficiency and group lending would be irrelevant.

[4] It can easily be proved that all entrepreneurs of a given type receive or do not receive a loan, i.e. there is no gain to expect from stochastic loan contracts. We focus also without loss of generality on symmetric solutions.

$$\pi_{21}(h-\underline{p}x_{21}-(1-\underline{p})y_{21})+\pi_{22}(h-\bar{p}x_{22}-(1-\bar{p})y_{22})\geq\frac{u}{\bar{p}}(\pi_{21}+\pi_{22}) \quad (2)$$

$$\pi_{11}(h-\underline{p}x_{11}-(1-\underline{p})y_{11})+\pi_{12}(h-\bar{p}x_{12}-(1-\bar{p})y_{12}) \\ \geq\pi_{11}(h-\underline{p}x_{21}-(1-\underline{p})y_{21})+\pi_{12}(h-\bar{p}x_{22}-(1-\bar{p})y_{22}) \quad (3)$$

$$\pi_{21}(h-\underline{p}x_{21}-(1-\underline{p})y_{21})+\pi_{22}(h-\bar{p}x_{22}-(1-\bar{p})y_{22}) \\ \geq\pi_{21}(h-\underline{p}x_{11}-(1-\underline{p})y_{11})+\pi_{22}(h-\bar{p}x_{12}-(1-\bar{p})y_{12}). \quad (4)$$

Furthermore, we have the wealth constraints:

$$x_{ij}\leq h \quad ; \quad y_{ij}\leq h \qquad \text{for all } i,\ j. \quad (5)$$

The bank's expected profit is, for a pair of entrepreneurs:

$$\pi_{11}[2\underline{p}^2x_{11}+2\underline{p}(1-\underline{p})y_{11}]+2\pi_{12}[\underline{p}\bar{p}(x_{12}+x_{21})+\underline{p}(1-\bar{p})y_{12}+(1-\underline{p})\bar{p}y_{21}] \\ +\pi_{22}[2\bar{p}^2x_{22}+2\bar{p}(1-\bar{p})y_{21}]-2r.$$

The average profit for the continuum of entrepreneurs is:

$$\pi_{11}\underline{p}[\underline{p}x_{11}+(1-\underline{p})y_{11}]+\pi_{12}[\underline{p}(\bar{p}x_{12}+(1-\bar{p})y_{12})+\bar{p}(\underline{p}x_{21}+(1-\underline{p})y_{21})] \\ +\pi_{22}\bar{p}[\bar{p}x_{22}+(1-\bar{p})y_{22}]-r. \quad (6)$$

We note that both in the objective function of the bank and in the constraints, the entrepreneurs' payments enter only through the expected terms $X_{11}=\underline{p}x_{11}+(1-\underline{p})y_{11}$, $X_{12}=\bar{p}x_{12}+(1-\bar{p})y_{12}$, $X_{21}=\underline{p}x_{21}+(1-\underline{p})y_{21}$, $X_{22}=\bar{p}x_{22}+(1-\bar{p})y_{22}$. The wealth constraints are obviously less constraining when $x_{ij}=y_{ij}$ for all i, j. Thus, we can rewrite the bank's program as:

$$\max_{(X_{ij})} \quad \pi_{11}\underline{p}X_{11}+\pi_{12}(\underline{p}X_{12}+\bar{p}X_{21})+\pi_{22}\bar{p}X_{22}-r \qquad i,j=1,2$$

s.t.

$$\pi_{11}(h-X_{11})+\pi_{12}(h-X_{12})\geq\frac{u}{\underline{p}}(\pi_{11}+\pi_{12}) \quad (7)$$

$$\pi_{21}(h-X_{21})+\pi_{22}(h-X_{22})\geq\frac{u}{\bar{p}}(\pi_{21}+\pi_{22}) \quad (8)$$

$$\pi_{11}(h-X_{11})+\pi_{12}(h-X_{12})\geq\pi_{11}(h-X_{21})+\pi_{12}(h-X_{22}) \quad (9)$$

$$\pi_{21}(h - X_{21}) + \pi_{22}(h - X_{22}) \geq \pi_{21}(h - X_{11}) + \pi_{22}(h - X_{12}) \tag{10}$$

$$X_{ij} \leq h \text{ for all } i,\ j. \tag{11}$$

We first show that if the correlation of types is high enough, the optimal pairing contracts are efficient and leave no rent to entrepreneurs.

Proposition 1. *When the correlation is high enough (π_{12} small enough), the optimal contracts of the bank are efficient.*

Proof. See Appendix B. □

The logic here is the one of Crémer and McLean (1988). Using the correlation of types, the bank can design rewards and penalties which, because entrepreneurs are risk neutral, both induce truthful revelation of types by entrepreneurs and make participation constraints binding. Indeed, the correlation of types doubles the degrees of freedom in the construction of revelation mechanisms. In our two-type environment, we have now four transfers X_{ij} instead of two for independent types. The incentive constraints can be satisfied with 2 degrees of freedom, so that we have 2 degrees of freedom left to saturate the Bayesian participation constraints.

Here, we find $X_{11} = X_{21} < X_{12} = X_{22}$, i.e. the payment of an entrepreneur is independent of his own type and greater when he is paired with a good type. We do not need to condition contracts on the production level of the partner, but only on announcements (and on the agent's production level because of limited wealth). Difficulties may arise from wealth constraints, but this does not occur for a correlation of types high enough.

To explore the effects of binding wealth constraints, we consider a special case of correlation which can be characterized by a single number (This case corresponds to the constraint $\pi_{11} = \pi_{22}$):

$$\pi_{11} = \pi_{22} = \frac{1}{2} - \varepsilon \quad ; \quad \pi_{12} = \pi_{21} = \varepsilon \quad ; \quad \rho > 0 \Leftrightarrow \varepsilon < \frac{1}{4}.$$

Then, Proposition 1 holds always if the wealth constraints are not binding, i.e. if (see Appendix B):

$$\varepsilon < \varepsilon^* = \frac{\underline{p}}{2(\bar{p} + \underline{p})}.$$

Proposition 2. *When $\pi_{11} = \pi_{22}$, the wealth constraint becomes binding for $\varepsilon > \varepsilon^*$. Then, the solution is*

$$X_{12} = X_{22} = h$$

$$X_{11} = X_{21} = h - \frac{u}{\underline{p}} \frac{\pi_{11} + \pi_{12}}{\pi_{11}}.$$

Then, the good type obtains a rent

$$R = \left(\frac{\varepsilon}{\frac{1}{2} - \varepsilon} \frac{\bar{p}}{\underline{p}} - 1 \right) u. \tag{12}$$

This opens the possibility that offering a contract not accepted by the bad type might be better. Indeed, the bank prefers to offer only a contract to the good type with no rent if

$$\underline{\pi}(\underline{p}h - r - u) < \bar{\pi}R, \tag{13}$$

i.e. if the expected profit made with type $\underline{p}$ entrepreneurs is less than the expected rent which must be given up to type $\bar{p}$ when a contract accepted by both types is offered.

When the correlation is high, it is possible by using two payments (one when the partner announces he is good, one when he announces he is bad) to discriminate between types and extract all the information rents. This is achieved despite the pooling nature of the optimal contract.

When the correlation of types is small, the bank asks the successful entrepreneur to pay his whole gain h when his partner announces he is a bad type and must give up a rent to the good type because it is not possible to exploit sufficiently the correlation of types. It shows one limit of yardstick competition[5] especially in developing countries where limited liability constraints are particularly severe.

4. Restricted contracts

4.1. Individual contracts

The bank may design individual contracts. An entrepreneur randomly chosen has a probability $\underline{\pi} = \pi_{11} + \pi_{12}$ (resp. $\bar{\pi} = \pi_{21} + \pi_{22}$) of being of type $\underline{p}$ (resp. $\bar{p}$).

The bank has two possible strategies, either to offer contracts which are accepted by both types of entrepreneurs (Regime 1), or to offer a contract which is only accepted by a good type (Regime 2). We obtain immediately:

Proposition 3. *If*

$$\underline{\pi}(\underline{p}h - r - u) > \bar{\pi} \frac{\bar{p} - \underline{p}}{\underline{p}} u,$$

Regime 1 holds. The bank offers a pooling contract which gives a loan to all entrepreneurs for a reimbursement $x = h - \frac{u}{\underline{p}}$ *when the project is successful.*
If

$$\underline{\pi}(\underline{p}h - r - u) < \bar{\pi} \frac{\bar{p} - \underline{p}}{\underline{p}} u.$$

[5] See Shleifer (1985) and Auriol and Laffont (1992).

Regime 2 holds. The bank offers a contract accepted only by type $\bar{p}$ *entrepreneurs with a reimbursement* $x = h - \frac{u}{\bar{p}}$ *when the project is successful.*

In Regime 1, an entrepreneur of type $\bar{p}$ has an expected rent

$$\bar{p}\left(h - \left(h - \frac{u}{\underline{p}}\right)\right) - u = \frac{\bar{p} - \underline{p}}{\underline{p}} u.$$

Regime 1 is preferred by the bank if the expected profit to be realized with type $\underline{p}$ entrepreneurs, $\pi(\underline{p}h - r - u)$, exceeds the expected rent,

$$\bar{\pi}\left(\frac{\bar{p} - \underline{p}}{\underline{p}}\right) u$$

that must be given up in Regime 1 to type $\bar{p}$ (in contrast to Regime 2 where no rent need to be given to these entrepreneurs), because of the presence of type $\underline{p}$.

In Regime 1, the allocation of loans is efficient, and the good type entrepreneurs are able to obtain a rent despite the monopolistic structure of banking. In Regime 2, the allocation of loans is inefficient since the valuable projects of type $\underline{p}$ are not financed, but the good type entrepreneurs obtain no rent.

Individual contracts are of course dominated by the optimal contracts. In the case $\pi_{11} = \pi_{22}$, from Eqs. (12) and (13), lending to both types occurs less often than with the efficient contract since for $\varepsilon < 1/4, \frac{\varepsilon}{1/2 - \varepsilon} < 1$.

4.2. Group lending contracts

A group lending contract is characterized, for any entrepreneur, by two possible payments when he is successful: X if his partner is also successful and Y if his partner is not successful.[6]

There is no incentive constraint and participation constraints write:

$$\pi_{11}\underline{p}[\underline{p}(h - X) + (1 - \underline{p})(h - Y)] + \pi_{12}\underline{p}[\bar{p}(h - X) + (1 - \bar{p})(h - Y)] \geq u(\pi_{11} + \pi_{12}) \tag{14}$$

$$\pi_{21}\bar{p}[\underline{p}(h - X) + (1 - \underline{p})(h - Y)] + \pi_{22}\bar{p}[\bar{p}(h - X) + (1 - \bar{p})(h - Y)] \geq u(\pi_{21} + \pi_{22}). \tag{15}$$

Using the notations of Section 3, we observe that:

$$X_{11} = X_{21} = \underline{p}X + (1 - \underline{p})Y \tag{16}$$

$$X_{12} = X_{22} = \bar{p}X + (1 - \bar{p})Y. \tag{17}$$

[6] Offering a menu of group lending contracts would bring no improvement.

Let $\underline{X}^* = X_{11} = X_{21}$ and $\bar{X}^* = X_{12} = X_{22}$ the payments in the optimal pairing contracts of Section 3. From Eqs. (16) and (17), we can implement those payments with

$$X = \frac{(1-\underline{p})\bar{X}^* - (1-\bar{p})\underline{X}^*}{\bar{p}-\underline{p}}$$

$$Y = \frac{\bar{p}\underline{X}^* - \underline{p}\bar{X}^*}{\bar{p}-\underline{p}}.$$

It remains to be seen when the limited liability constraints become binding. In the example $\pi_{11} = \pi_{21} = (1/2) - \varepsilon$, $\pi_{12} = \pi_{21} = \varepsilon$, Y is always less than h and

$$X \leq h \quad \text{iff} \quad \varepsilon < \frac{1}{2}\left(1 - \frac{1}{\underline{p}+\bar{p}}\right).$$

As expected, this condition is always more stringent than the one obtained for the optimal contract since

$$\frac{\underline{p}}{2(\bar{p}+\underline{p})} > \frac{1}{2}\left(1 - \frac{1}{\underline{p}+\bar{p}}\right).$$

When ε is too large, $X = h$ and

$$Y = h - \frac{u}{\underline{p}}\frac{1}{(1-\underline{p}) + 2\varepsilon(\underline{p}-\bar{p})}$$

with a rent for the good type.[7]

Proposition 4. *The optimal group lending contracts are efficient if correlation is high enough but are not always optimal.*

A striking feature is that contrary to the practice of Grameen Banks, the payments required from a successful entrepreneur are higher when his partner is successful than when he fails.[8] This is because we have assumed a positive correlation of types. Payments must differ to use the correlation for rent extraction. The positive correlation implies that it is better to extract more in more likely events. To rationalize in our model with adverse selection the practice of Grameen Bank's contracts for which an additional payment is required when a partner fails, we would need to assume negative correlation of types. We do not argue that negative correlation is a good rationalization of Grameen Bank contracts. We rather draw the conclusion that, for the type of model considered here, such

[7] Of course, loans to good types only occur in similar circumstances as in Section 3, Eq. (13) with the proper definition of the rent.

[8] Note that this feature creates an incentive for an entrepreneur to make his partner's project unsuccessful.

rationalization must be looked for elsewhere for example as an attempt to solve moral hazard or enforcement problems.[9]

Alternatively, we may consider a different model with adverse selection. So far, we have consider what Ghatak (2000) calls the De Meza and Webb (1987)[10] setup in which entrepreneurs differ in their probability of success, but obtain the same payoff h when they succeed. The advantage of this setup is that the observability of success does not enable the bank to infer the type of the entrepreneur.

He also considers (as well as Armendariz de Aghion and Gollier, 2000) the Stiglitz and Weiss (1981) setup where expected revenues are identical $\underline{p}\underline{h}=\bar{p}\bar{h}$, which requires $\bar{h}<\underline{h}$, i.e. a good type (in terms of probability of success) obtains less when he succeeds. To avoid the inference of the type ex post, the authors must make the additional (and questionable) assumption that the bank observes if the entrepreneur succeeds but not proceeds $\underline{h}$ or $\bar{h}$. As Ghatak (2000) shows, the results obtained are then very different. In Appendix C, we briefly adapt our analysis to this setup to explain why one can then conclude that the repayment can be greater if the partner fails than if he succeeds (let us call that the "Grameen Bank property").

Considering the more general case where $\underline{h}=h+\delta$, $\bar{h}=h$ we see that this result holds[11] if

$$\delta > \frac{u(\bar{p}-\underline{p})}{\underline{p}\bar{p}},$$

which is always true in the Stiglitz-Weiss case $\left(\delta=\frac{h(\bar{p}-\underline{p})}{\underline{p}}\right)$.

The intuition is as follows. Payments are higher when the partner fails if and only if one can extract more in the optimal mechanism from an entrepreneur paired with a bad type than from an entrepreneur paired with a good type. This last result is purely driven by the desire to make binding both participation constraints. When both types have the same expected revenue $\underline{p}(h+\delta)=\bar{p}h$, one can extract more when paired with a bad type than a good type because:

- for a good type who reimburses more often, this is a less likely event because of positive correlation;
- for a bad type, it is on the contrary more likely, but it is compensated by the fact that he pays less often.

When the two types have the same payoff when they succeed, this is not possible, because the more likely payment he has to make breaks his participation constraint if the one of the good type is binding.

Thus, the driving force is not that a higher payment, when paired with an unsuccessful partner, weakens the incentive constraint through a self-selection mechanism, since incentive constraints are irrelevant here.

Suppose one imposes, in the De Meza–Webb framework, that payment, if the partner fails, is higher, than if he succeeds and that we allow for a menu of group lending

[9] See Laffont and Rey (2000) for moral hazard and Laffont and N'Guessan (2001) for enforcement.

[10] See also Besanko and Thakor (1987).

[11] See Appendix C.

contracts X_1, Y_1 for type $\underline{p}$ and X_2, Y_2 for type $\bar{p}$ with $Y_1 \geq X_1$ and $Y_2 \geq X_2$. Then, we obtain:

Proposition 5. *If we allow for a menu of group lending contracts constrained to $Y_i \geq X_i$, $i = 1, 2$, then the optimal menu is such that $X_1 = Y_1$ and $Y_2 > X_2$.*

Proof. See Appendix D. □

Under the constraint of the Grameen Bank property, the menu of group lending contracts yields a self-selection effect since good types are willing to pay more if their partner fails, because it is less likely due to the positive correlation.

Thus, it appears that the self-selection effect discussed in the literature (Ghatak, 2000; Armendariz de Aghion and Aghion, 2000) occurs in our monopolistic setting only when group lending contracts are constrained to be of the "Grameen Bank" type. It is therefore not a justification of Grameen Bank contracts, but the reverse. The self-selection effect is a second-best response to the "Grameen Bank" constraint.

5. Collusion under complete information

Let us first assume that entrepreneurs may collude when they play the revelation mechanism offered by the bank but after having accepted the offer of the bank. Accordingly, the participation constraints remain the interim individual participation constraints. Furthermore, we assume that entrepreneurs always share their private information after having accepted the bank's contract, and that a ringmaster organizes the collusion. More precisely, we have the following timing:[12]

Entrepreneurs obtain their private information	Bank offers mechanism G	Entrepreneurs accept or reject G	Entrepreneurs share information	An uninformed ringmaster offers a collusion contract S	$G \circ S$ is played

Because entrepreneurs share information, individual incentive constraints are dominant strategy incentive constraints

$$h - \underline{p}x_{11} - (1 - \underline{p})y_{11} \geq h - \underline{p}x_{21} - (1 - \underline{p})y_{21} \tag{18}$$

$$h - \bar{p}x_{12} - (1 - \bar{p})y_{12} \geq h - \bar{p}x_{22} - (1 - \bar{p})y_{22} \tag{19}$$

$$h - \underline{p}x_{21} - (1 - \underline{p})y_{21} \geq h - \underline{p}x_{11} - (1 - \underline{p})y_{11} \tag{20}$$

$$h - \bar{p}x_{22} - (1 - \bar{p})y_{22} \geq h - \bar{p}x_{12} - (1 - \underline{p})y_{12}. \tag{21}$$

[12] We are not taking into account the financial constraints in the collusive side-contracts proposed by the ringmaster. This exaggerates the threat of collusion, but it is of no relevance for the results we present below. Furthermore, it can be justified if investors have hidden wealth they can use in their side-contracting (but could not be used as collateral).

When internal transfers are available within the coalition, collusion-proof constraints simplify to, for a pair (1,1):

$$2(h-\underline{p}x_{11}-(1-\underline{p})y_{11})\geq 2h-\underline{p}x_{ij}-(1-\underline{p})y_{ij}-\underline{p}x_{ji}-(1-\underline{p})y_{ji} \quad \text{for all } i,\ j, \tag{22}$$

for a pair (1,2):

$$\underline{p}(h-\bar{p}x_{12}-(1-\bar{p})y_{12})+\bar{p}(h-\underline{p}x_{21}-(1-\underline{p})y_{21})\geq\underline{p}(h-\bar{p}x_{ij}-(1-\bar{p})y_{ij}) + \bar{p}(h-\underline{p}x_{ji}-(1-\underline{p})y_{ji}) \quad \text{for all } i,\ j, \tag{23}$$

for a pair (2,2):

$$2(h-\bar{p}x_{22}-(1-\bar{p})y_{22})\geq 2h-\bar{p}x_{ij}-(1-\bar{p})y_{ij}-\bar{p}x_{ji}-(1-\bar{p})y_{ji} \quad \text{for all } i,\ j. \tag{24}$$

If we proceed as in Section 3 and do not distinguish payments according to the success or failure of the partner, i.e. $X_{ij}=x_{ij}=y_{ij}$ for all i, j, incentive constraints imply

$$X_{11}=X_{21}=X_{22}=X_{12}.$$

We are then back immediately to the individual contracts of Section 4.1.

Suppose, on the contrary, that we keep the flexibility of $x_{ij}\neq y_{ij}$.

Dominant strategy incentive constraints imply

$$\underline{p}x_{11}+(1-\underline{p})y_{11}=\underline{p}x_{21}+(1-\underline{p})y_{21} \tag{25}$$

$$\bar{p}x_{22}+(1-\bar{p})y_{22}=\bar{p}x_{12}+(1-\bar{p})y_{12}, \tag{26}$$

and the collusion-proof constraints reduce to

$$\underline{p}x_{11}+(1-\underline{p})y_{11}\leq\underline{p}x_{12}+(1-\underline{p})y_{12} \tag{27}$$

$$\bar{p}x_{22}+(1-\bar{p})y_{22}\leq\bar{p}x_{21}+(1-\bar{p})y_{21} \tag{28}$$

$$\underline{p}x_{21}+(1-\underline{p})y_{21}\leq\underline{p}x_{22}+(1-\underline{p})y_{22} \tag{29}$$

$$\bar{p}x_{12}+(1-\bar{p})y_{12}\leq\bar{p}x_{11}+(1-\bar{p})y_{11}, \tag{30}$$

with the wealth constraints:

$$x_{ij}\leq h \quad \text{for all } i,\ j$$

$$y_{ij}\leq h \quad \text{for all } i,\ j.$$

With the interim participation constraints and the incentive constraints, we have eight constraints and also eight variables. Furthermore, we have the wealth constraints.

Imposing

$$x_{11} = x_{12} = x_{21} = x_{22} = X$$

$$y_{11} = y_{12} = y_{21} = y_{22} = Y$$

enables us to satisfy all individual and coalition incentive constraints, and also to extract all the rents if

$$h - \frac{\underline{p}\pi_{11} + \bar{p}\pi_{12}}{\pi_{11} + \pi_{12}} X - \frac{(1-\underline{p})\pi_{11} + (1-\bar{p})\pi_{12}}{\pi_{11} + \pi_{12}} Y = \frac{u}{\underline{p}}$$

$$h - \frac{\underline{p}\pi_{21} + \bar{p}\pi_{22}}{\pi_{21} + \pi_{22}} X - \frac{(1-\underline{p})\pi_{21} + (1-\bar{p})\pi_{22}}{\pi_{21} + \pi_{22}} Y = \frac{u}{\bar{p}}.$$

The determinant of this system is $(\underline{p} - \bar{p})\rho$ which is non-null as soon as there is some correlation. We can find X and Y which solve the system.

In the case $\pi_{11} = \pi_{22} = (1/2) - \varepsilon$, $\pi_{12} = \pi_{21} = \varepsilon$, we obtain:

$$X = h - \frac{u}{\underline{p}\bar{p}} \left[\frac{(1-2\varepsilon)(\bar{p} + \underline{p}) - 1}{1 - 4\varepsilon} \right]$$

$$Y = h - \frac{u}{\underline{p}\bar{p}} \left[\frac{(1-2\varepsilon)(\bar{p} + \underline{p})}{1 - 4\varepsilon} \right] < h.$$

with

$$X < h \text{ if } \varepsilon < \frac{1}{2}\left(1 - \frac{1}{\underline{p} + \bar{p}}\right).$$

We obtain (for $\underline{p} + \bar{p} > 1$):

Proposition 6. *The optimal collusion-proof contract is the optimal group lending contract if correlation is high enough.*[13]

The intuition for this result is simply the following: announcement contracts are not robust to collusion, while contracts which depend on the production of the partner are robust to collusion (since we assumed that production is verifiable). Furthermore, for a correlation high enough, the group lending contracts yield the first best. Therefore, they are then the optimal collusion-proof contracts.

Note that the optimal collusion-proof contract is not here what would result from the optimal contract of Section 3 with collusion. Indeed, entrepreneurs would then always claim

[13] In the limit for $\varepsilon = 0$, it may seem that it is as if the bank was facing a single agent. This is true for incentive constraints, but we have here two participation constraints that the bank can saturate because, by observing the success or failure of both projects, it still has 2 degrees of freedom.

that they are both bad types (since from Appendix B we notice that $X_{12}=X_{22}>X_{11}=X_{21}$). They would always pay X_{11}, and therefore, the pair of good types would have a rent contrary to what is achieved in the group lending contract for a high correlation.

Group lending contracts have been presented in the literature (Ghatak, 2000; Armendariz de Aghion and Gollier, 2000) as useful to allow some discrimination between types. We have shown that their value for discrimination is limited and that, for this purpose, they are dominated by contracts which vary payments as a function of the agent's announcements. However, these latter contracts are not collusion-proof if agents can collude when they play the announcement game. On the contrary, the group lending contracts are robust to this type of collusion while still allowing some discrimination. This is achieved by exploiting the correlation of types in the uncertainty on final production.

6. Extended mechanisms

We assume now that the mechanism asks from agents the whole vector of types once they have shared their information. Any deviation from the sending of the same messages by the two agents is punished. We are left with the collusion-proof constraints as incentive constraints.

Suppose one does not distinguish payments according to the success or failure of the partner. We call such mechanisms unconditional extended revelation mechanisms. These constraints reduce to

$$X_{11}=X_{22}\leq\frac{X_{12}+X_{21}}{2}$$

$$\frac{\underline{p}X_{12}+\bar{p}X_{21}}{2}\leq X_{11}=X_{22}.$$

Proposition 7. *Unconditional extended revelation mechanisms cannot be collusion proof and efficient.*

See Appendix E for the proof. The intuition of this result is that, for a coalition, we have three types ($\underline{p}\underline{p}$, $\underline{p}\bar{p}$, $\bar{p}\bar{p}$) and three revelant incentive constraints at least, and also a participation constraint for each agent, hence five constraints, and only 4 degrees of freedom. The added flexibility of unconditional extended revelation mechanisms is not enough to achieve efficiency, while simple group lending contract do achieve efficiency when the correlation is high enough. (Proposition 4 remains valid with collusion when extended mechanisms are used.) Using group lending extended mechanisms will increase the range of parameters for which efficiency is achieved. We can safely conclude that group lending contracts remain useful to deal with collusion in this extended framework.

7. Conclusion

We have considered a simple model of lending to borrowers who have private information on the quality of their investment project and who have limited liability in order to make two points. On the one hand, group lending contracts are a particular way of practicing a subtle type of discrimination and constitute a powerful tool of rent extraction when types are correlated. However, they are not optimal instruments. On the other hand, we have shown that group lending contracts are interesting to extract rents when collusive behavior is possible.

These results should be robust to more general situations with loans of variable sizes, endogenous grouping and to alternative formulations of the agency problem with adverse selection faced by the bank. We leave for further research a more detailed analysis of the optimal collusion-proof contracts, when collusion takes places under asymmetric information with limited liability constraints, taking into account, in particular, the design of side-contracts.

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Appendix A. Bayesian incentive and participation constraints

Consider type $\underline{p}$. His posterior probabilities about the type of his partner are:

$$\frac{\pi_{11}}{\pi_{11}+\pi_{12}} \quad \text{for type } \underline{p}$$

and

$$\frac{\pi_{12}}{\pi_{11}+\pi_{12}} \quad \text{for type } \bar{p}.$$

His (Bayesian) participation constraint is:

$$\frac{\pi_{11}}{\pi_{11}+\pi_{12}}\underline{p}(h-\underline{p}x_{11}-(1-\underline{p})y_{11})+\frac{\pi_{12}}{\pi_{11}+\pi_{12}}\underline{p}(h-\bar{p}x_{12}-(1-\bar{p})y_{12})\geq u$$

hence Eq. (1) and similarly for Eq. (2).

Indeed, with probability $\underline{p}$, he obtains h. With probability $\frac{\pi_{11}}{\pi_{11}+\pi_{12}}$, he will be paired with a type $\underline{p}$ partner. In this case, his partner succeeds with probability $\underline{p}$ and he must reimburse x_{11}. His partner fails with probability $(1-\underline{p})$ and he must then reimburse y_{11}. With probability $\frac{\pi_{12}}{\pi_{11}+\pi_{12}}$, he will be paired with a type $\bar{p}$ partner who succeeds with probability $\bar{p}$ (in which case, he pays x_{12}) and fails with probability $1-\bar{p}$ (in which case he pays y_{12}) and similarly for type $\bar{p}$.

His (Bayesian) incentive constraint is

$$\frac{\pi_{11}}{\pi_{11}+\pi_{12}}\underline{p}(h-\underline{p}x_{11}-(1-\underline{p})y_{11})+\frac{\pi_{12}}{\pi_{11}+\pi_{12}}\underline{p}(h-\bar{p}x_{12}-(1-\bar{p})y_{12})$$
$$\geq\frac{\pi_{11}}{\pi_{11}+\pi_{12}}\underline{p}(h-\underline{p}x_{21}-(1-\underline{p})y_{21})+\frac{\pi_{12}}{\pi_{11}+\pi_{12}}\underline{p}(h-\bar{p}x_{22}-(1-\bar{p})y_{22})$$

hence, Eq. (3) and similarly for Eq. (4).

Appendix B. Proof of Propositions 1 and 2

Proposition 1:

Constraints (7)–(10) are binding if:

$$\begin{bmatrix}-\pi_{11} & \pi_{12} & 0 & 0\\ 0 & 0 & -\pi_{21} & -\pi_{22}\\ -\pi_{11} & -\pi_{12} & \pi_{11} & \pi_{12}\\ \pi_{21} & \pi_{22} & -\pi_{21} & -\pi_{22}\end{bmatrix}\begin{bmatrix}X_{11}\\ X_{12}\\ X_{21}\\ X_{22}\end{bmatrix}=\begin{bmatrix}(\pi_{11}+\pi_{12})(\frac{u}{\underline{p}}-h)\\ (\pi_{21}+\pi_{22})(\frac{u}{\bar{p}}-h)\\ 0\\ 0\end{bmatrix}=\begin{bmatrix}a\\ b\\ 0\\ 0\end{bmatrix}$$

The determinant of this system is $\Delta=-\rho^2$.

Solving the system we obtain:

$$X_{11}=X_{21}=\frac{b\pi_{12}-a\pi_{22}}{\rho}=h-\frac{u}{\underline{p}}-\frac{u}{\rho}\pi_{12}(\pi_{12}+\pi_{22})\frac{\bar{p}-\underline{p}}{\bar{p}\underline{p}}$$

$$X_{12}=X_{22}=\frac{a\pi_{21}-b\pi_{11}}{\rho}=h-\frac{u}{\bar{p}}+\frac{u}{\rho}\pi_{21}(\pi_{11}+\pi_{12})\frac{\bar{p}-\underline{p}}{\bar{p}\underline{p}}.$$

These payments satisfy the limited liability constraint if π_{12} is close enough to zero. Then, both entrepreneurs are indifferent between lying or telling the truth about their type and no rent is given up. They are rewarded if their partner is bad and punished if he is good.

Proposition 2:

As π_{12} increases, we reach the boundary h for X_{12}.

For example, if $\pi_{11}=\pi_{22}=(1/2)-\varepsilon$ and $\pi_{12}=\pi_{21}=\varepsilon$, the boundary is reached for

$$\varepsilon^*=\frac{\underline{p}}{2(\bar{p}+\underline{p})}.$$

For $\varepsilon > \varepsilon^*$, the solution entails $X_{12} = X_{22} = h$, hence, the constraints:

$$\pi_{11}(h - X_{11}) \geq \frac{u}{\underline{p}}(\pi_{11} + \pi_{12}) \tag{A.1}$$

$$\pi_{21}(h - X_{21}) \geq \frac{u}{\bar{p}}(\pi_{21} + \pi_{22}) \tag{A.2}$$

$$\pi_{11}(h - X_{11}) \geq \pi_{11}(h - X_{21}) \tag{A.3}$$

$$\pi_{21}(h - X_{21}) \geq \pi_{21}(h - X_{11}). \tag{A.4}$$

From Eqs. (A.3) and (A.4), $X_{11} = X_{21} = X$.

We can expect the bad type's participation constraint to be binding

$$X = h - \frac{u}{\underline{p}} \frac{\pi_{11} + \pi_{12}}{\pi_{11}}$$

with an expected rent for the good type:

$$\left[\frac{\pi_{21}(\pi_{11} + \pi_{12})}{\pi_{11}(\pi_{21} + \pi_{22})} \frac{\bar{p}}{\underline{p}} - 1 \right] u.$$

In the special case, this is positive if ε is larger than ε^*.

Appendix C. Characterization of the condition for the "Grameen Bank property" to hold

Consider the more general model in which type $\bar{p}$ obtains h when he succeeds and type $\bar{p}$ obtains $h + \delta$. If $\delta = 0$, we have the De Meza and Webb (1987) model; if δ is such that ($\bar{p}h = \underline{p}(h + \delta)$), we have the Stiglitz and Weiss (1981) model.

The analysis of optimal contracts and optimal individual contracts is exactly the same as in the text. Consider the optimal contract (X_{11},X_{12},X_{21},X_{22}) for which participation and incentive constraints bind. They solve now (see Appendix B):

$$\begin{bmatrix} -\pi_{11} & -\pi_{12} & 0 & 0 \\ 0 & 0 & -\pi_{21} & -\pi_{22} \\ -\pi_{11} & -\pi_{12} & \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} & -\pi_{21} & -\pi_{22} \end{bmatrix} \begin{bmatrix} X_{11} \\ X_{12} \\ X_{21} \\ X_{22} \end{bmatrix} = \begin{bmatrix} (\pi_{11} + \pi_{12})(\frac{u}{\underline{p}} - h - \delta) \\ (\pi_{21} + \pi_{22})(\frac{u}{\bar{p}} - h) \\ 0 \\ 0 \end{bmatrix},$$

i.e.

$$\underline{X}^* = X_{11} = X_{21} = \frac{1}{\rho}\left[(\pi_{21}+\pi_{22})\left(\frac{u}{\bar{p}} - h\right)\pi_{12} - (\pi_{11}+\pi_{12})\left(\frac{u}{\underline{p}} - h - \delta\right)\pi_{22}\right]$$

$$\bar{X}^* = X_{12} = X_{22} = \frac{1}{\rho}\left[(\pi_{11}+\pi_{12})\left(\frac{u}{\underline{p}} - h - \delta\right)\pi_{21} - (\pi_{22}+\pi_{12})\left(\frac{u}{\underline{p}} - h\right)\pi_{11}\right].$$

The associated group lending contract is such that (see Section 4.2):

$$X = \frac{(1-\underline{p})\bar{X}^* - (1-\bar{p})\underline{X}^*}{\bar{p}-\underline{p}} \quad ; \quad Y = \frac{\bar{p}\underline{X}^* - \underline{p}\bar{X}^*}{\bar{p}-\underline{p}}.$$

Note that

$$Y - X = \frac{\underline{X}^* - \bar{X}^*}{\bar{p}-\underline{p}}.$$

Thus, the "Grameen Bank property" holds if one can extract more when paired with a bad type than with a good type ($\underline{X}^* > \bar{X}^*$), which reduces to

$$\delta > \frac{u(\bar{p}-\underline{p})}{\underline{p}\bar{p}}.$$

In particular, for the Stiglitz–Weiss setup for which $\delta = \frac{h(\bar{p}-\underline{p})}{\underline{p}}$, this inequality becomes $\bar{p}h > u$ which holds since all projects are socially valuable. For the De Meza–Webb setup ($\delta = 0$), it cannot hold.

Appendix D. Proof of Proposition 5

Allowing for a menu of group lending contracts X_1, Y_1, X_2, Y_2 with the constraint $Y_1 \geq X_1$, $Y_2 \geq X_2$, the bank's problem can be rewritten (with $\Delta_1 = Y_1 - X_1$; $\Delta_2 = Y_2 - X_2$):

$$\max(2\pi_{11} + \pi_{12})\underline{p}Y_1 + (2\pi_{22} + \pi_{21})\bar{p}Y_2 - (2\pi_{11}\underline{p}^2 + \pi_{12}\underline{p}\bar{p})\Delta_1$$
$$- (2\pi_{22}\bar{p}^2 + \pi_{21}\underline{p}\bar{p})\Delta_2,$$

s.t.

$$(\Delta_2 - \Delta_1)a + Y_1 - Y_2 \geq 0$$

$$(\Delta_1 - \Delta_2)b + Y_2 - Y_1 \geq 0$$

$$b\Delta_1 - Y_1 \geq c$$

$$a\Delta_2 - Y_2 \geq d$$

$$Y_1 \leq h, \quad Y_2 \leq h, \quad \Delta_1 \geq 0, \quad \Delta_2 \geq 0,$$

for

$$a = \frac{\pi_{21}\underline{p} + \pi_{22}\bar{p}}{\pi_{21} + \pi_{22}}$$

$$b = \frac{\pi_{11}\underline{p} + \pi_{12}\bar{p}}{\pi_{11} + \pi_{12}}$$

$$c = -h + \frac{u}{\underline{p}}$$

$$d = -h + \frac{u}{\bar{p}}.$$

Then, it is easy to see that the optimal solution entails

$$Y_2 = h \text{ and } X_2 = h - \frac{u}{\underline{p}a}$$

$$\Delta_1 = 0 \text{ with } X_1 = Y_1 = -c$$

$$Y_2 = h \text{ and } X_2 = h - \frac{u}{a\underline{p}}.$$

Appendix E. Proof of Proposition 7

Let us denote $x = X_{11} = X_{22}$.
To bind both participation constraints, we need

$$X_{12} = -\frac{\pi_{11}}{\pi_{12}}x + \frac{\pi_{11} + \pi_{12}}{\pi_{12}}\left(h - \frac{u}{\underline{p}}\right)$$

$$X_{21} = -\frac{\pi_{22}}{\pi_{21}}x + \frac{\pi_{21} + \pi_{22}}{\pi_{21}}\left(h - \frac{u}{\bar{p}}\right).$$

The constraint

$$\frac{X_{12} + X_{21}}{2} \geq X_{22} = X_{11}$$

writes now as

$$x \leq h - (\pi_{11} + \pi_{12})\frac{u}{\underline{p}} - (\pi_{21} + \pi_{22})\frac{u}{\bar{p}}.$$

The constraint

$$\frac{\underline{p}X_{12} + \bar{p}X_{21}}{\underline{p} + \bar{p}} \leq X_{11} = X_{22}$$

writes now as

$$x \geq h - \frac{u}{\underline{p}(\pi_{11} + \pi_{12}) + \bar{p}(\pi_{21} + \pi_{22})}.$$

These inequalities are compatible if

$$\underline{p}^2 + \bar{p}^2 + \underline{p}\bar{p}\left[\frac{(\pi_{11} + \pi_{12})^2 + (\pi_{21} + \pi_{22})^2 - 1}{(\pi_{11} + \pi_{12})(\pi_{22} + \pi_{21})}\right] \leq 0.$$

However, the left-hand side equals

$$\underline{p}^2 + \bar{p}^2 - 2\underline{p}\bar{p} = (\bar{p} - \underline{p})^2 > 0 \quad \text{if } \bar{p} > \underline{p}.$$

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